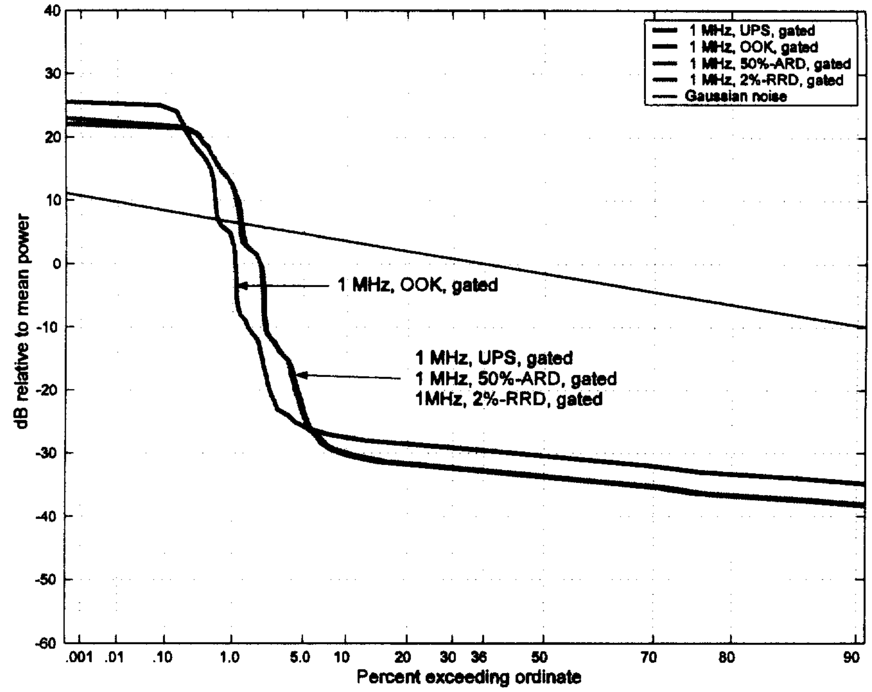

Figure C.3.17. APDs of 1-MHz PRF, gated UWB signals measured in a 20-MHz bandwidth.

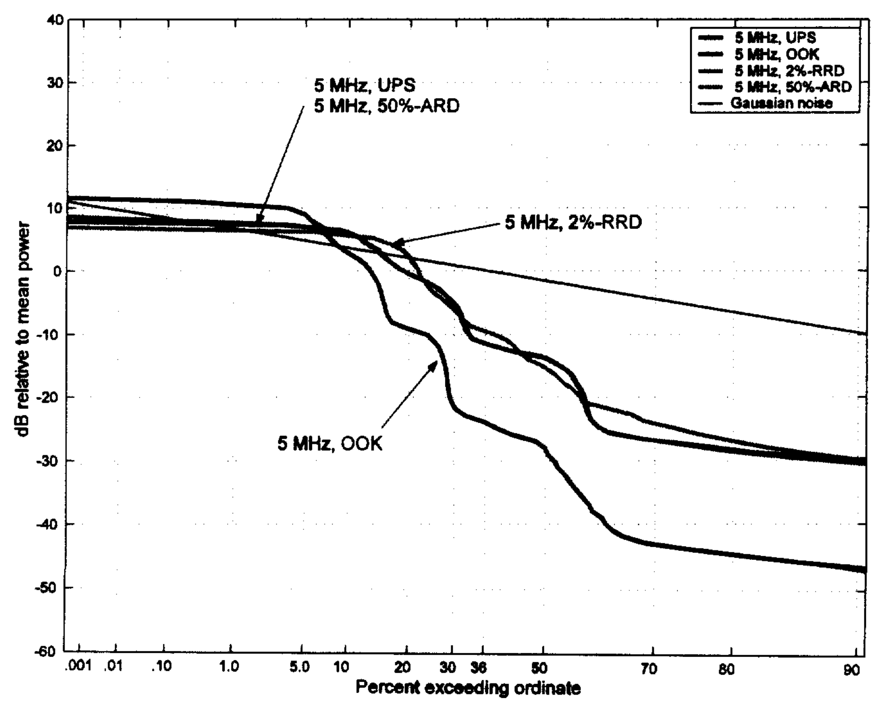

Figure C.3.18. APDs of 5-MHz PRF, non-gated UWB signals measured in a 20-MHz bandwidth.

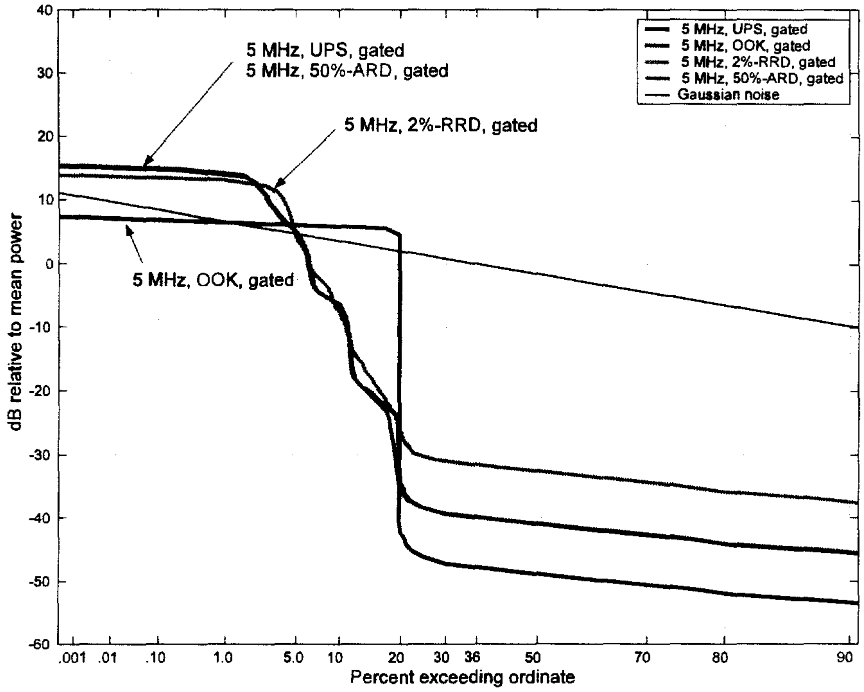

Figure C.3.19. APDs of 5-MHz PRF, gated UWB signals measured in a 20-MHz bandwidth.

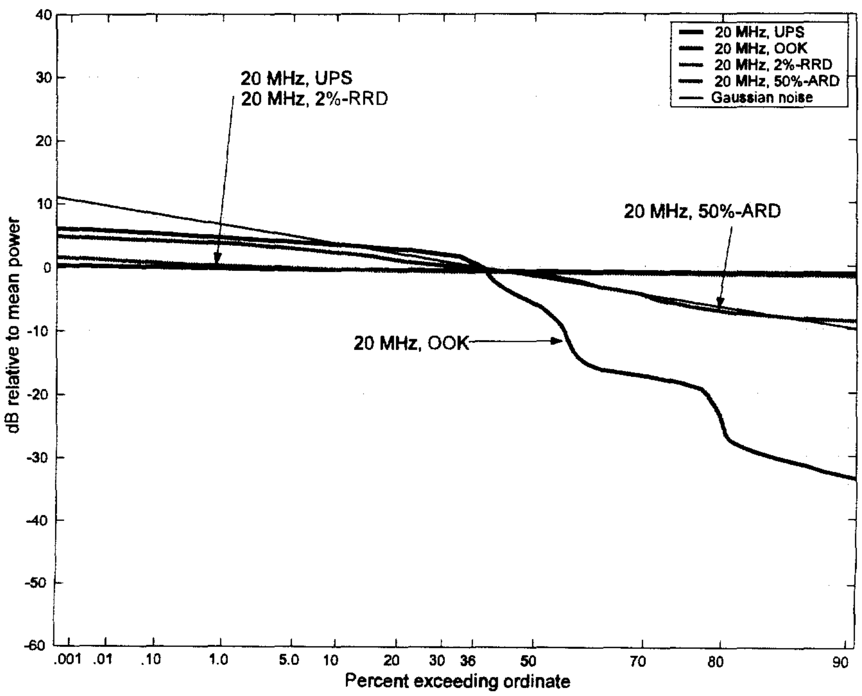

Figure C.3.20. APDs of 20-MHz PRF, non-gated UWB signals measured in a 20 MHz bandwidth.

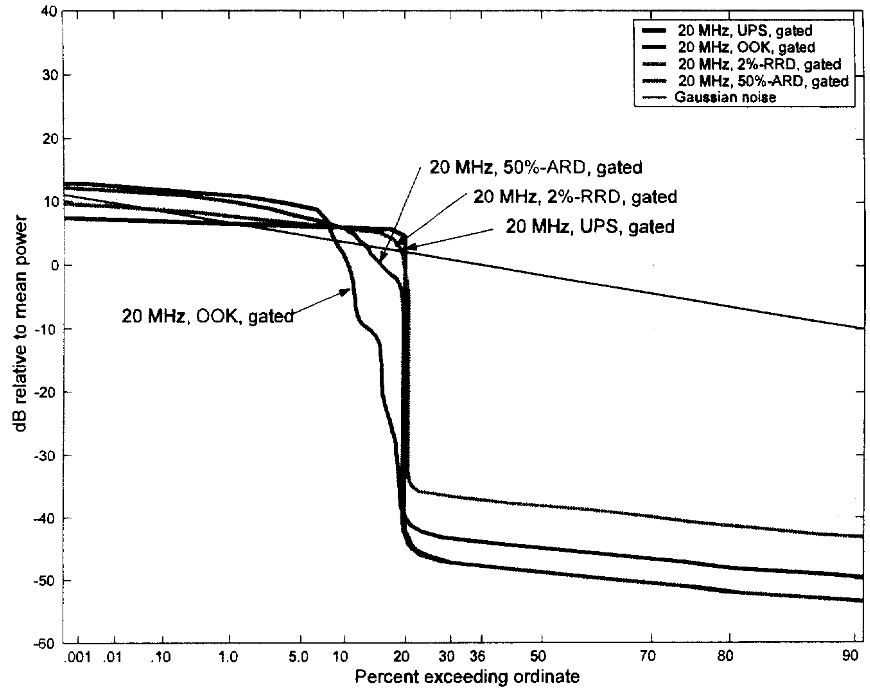

Figure C.3.21. APDs of 20-MHz PRF, gated UWB signals measured in a 20-MHz bandwidth.

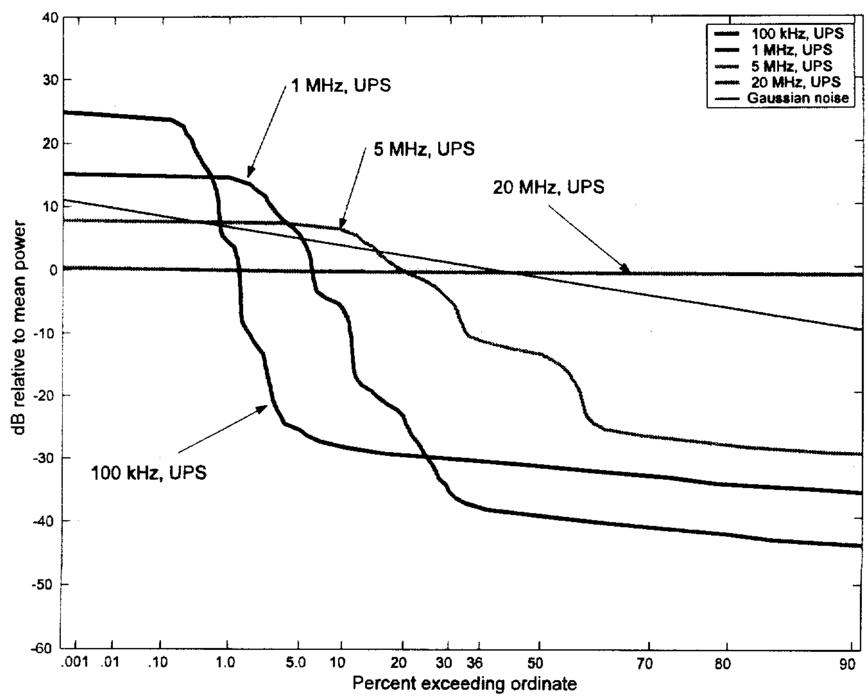

Figure C.3.22. APDs of UPS, non-gated UWB signals measured in a 20-MHz bandwidth.

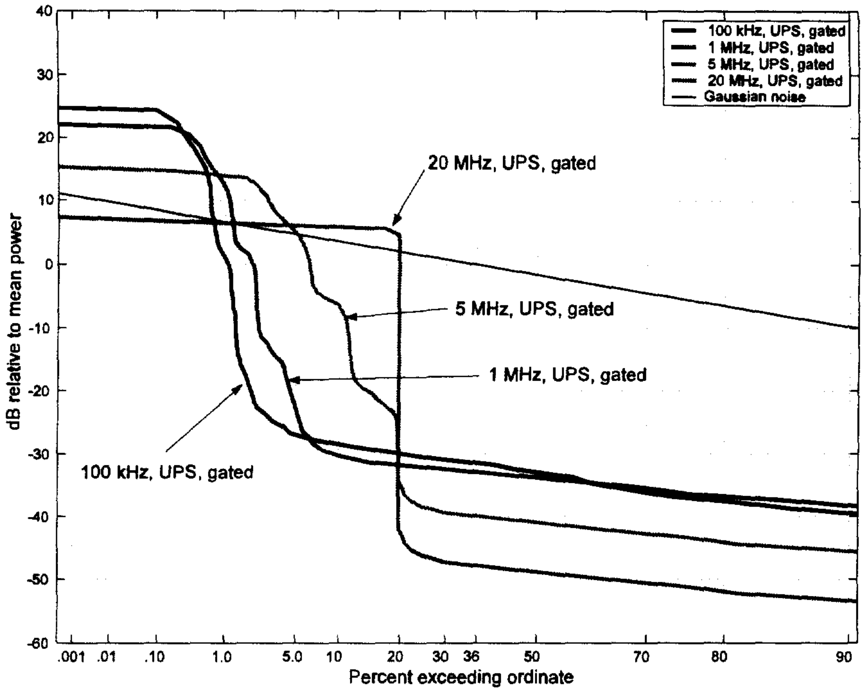

Figure C.3.23. APDs of UPS, gated UWB signals measured in a 20-MHz bandwidth.

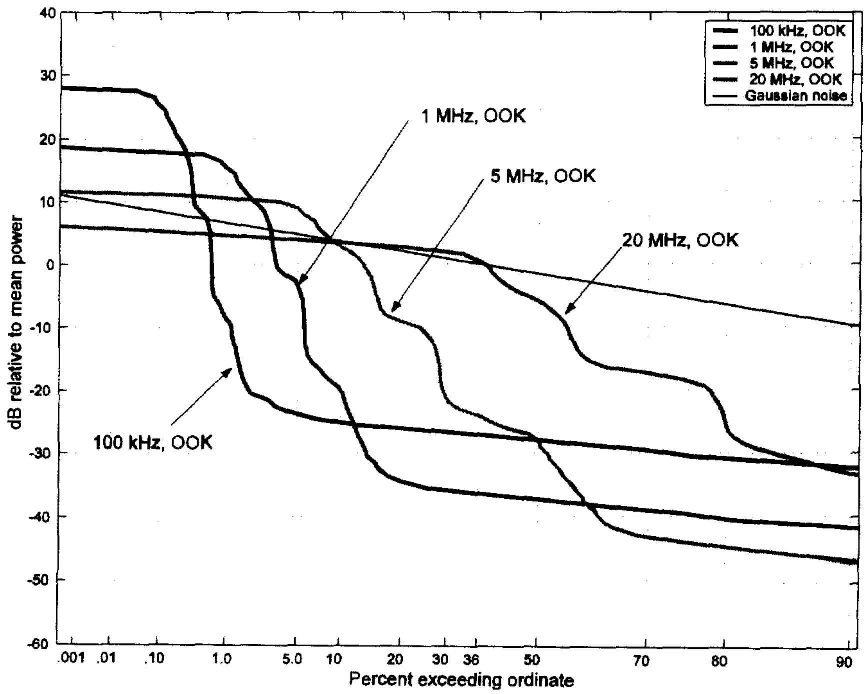

Figure C.3.24. APDs of OOK, non-gated UWB signals measured in a 20-MHz bandwidth.

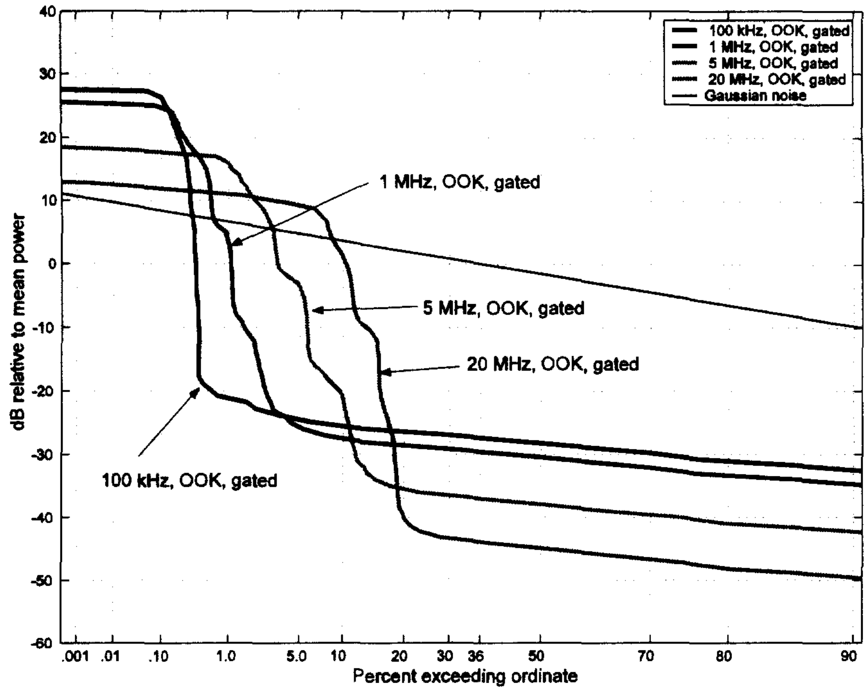

Figure C.3.25. APDs of OOK, gated UWB signals measured in a 20-MHz bandwidth.

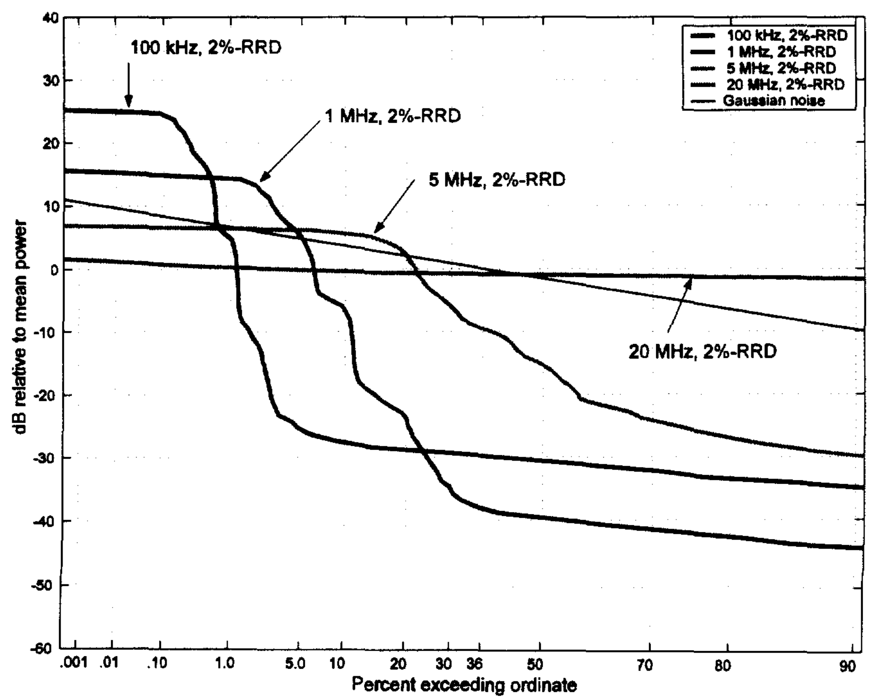

Figure C.3.26. APDs of 2%-RRD, non-gated UWB signals measured in a 20-MHz bandwidth.

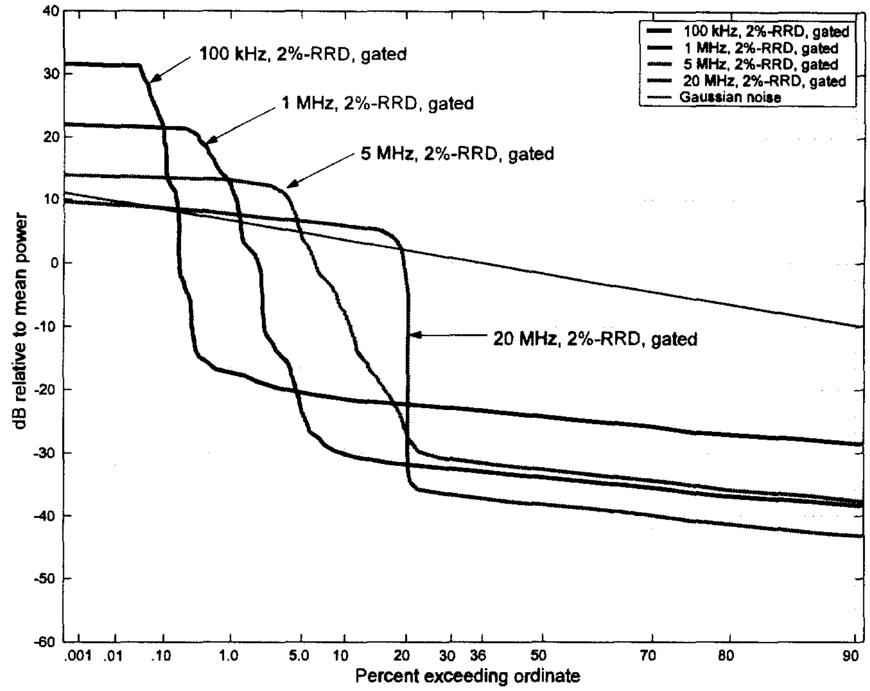

Figure C.3.27. APDs of 2%-RRD, gated UWB signals measured in a 20-MHz bandwidth.

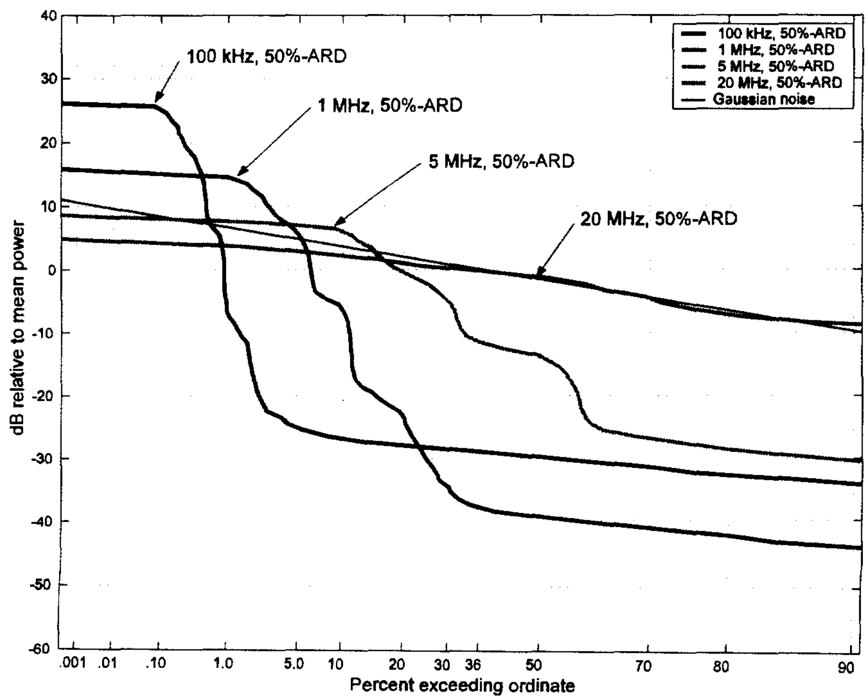

Figure C.3.28. APDs of 50%-ARD, non-gated UWB signals measured in a 20-MHz bandwidth.

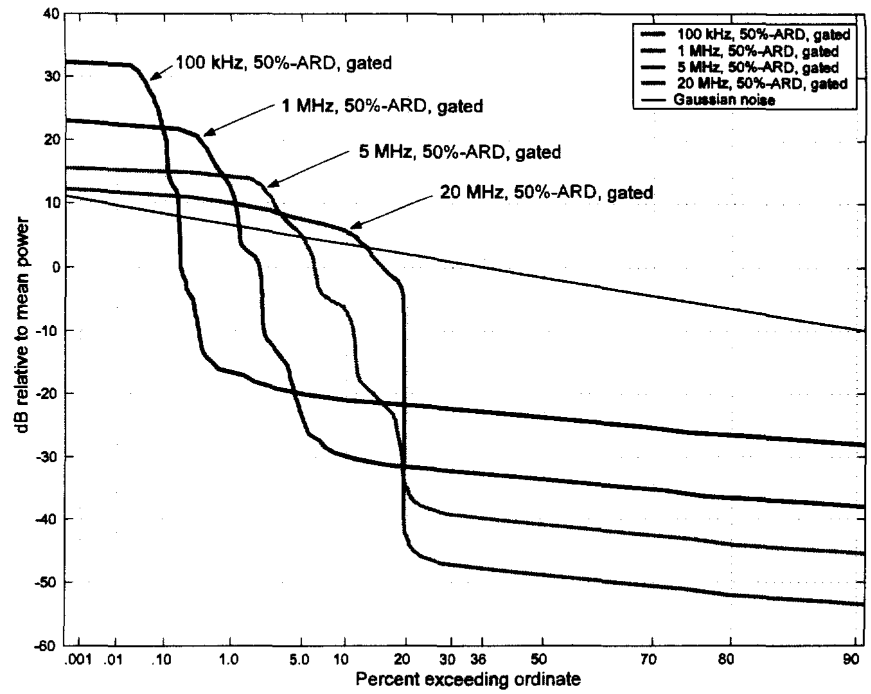

Figure C.3.29. APDs of 50%-ARD, gated UWB signals measured in a 20-MHz bandwidth.

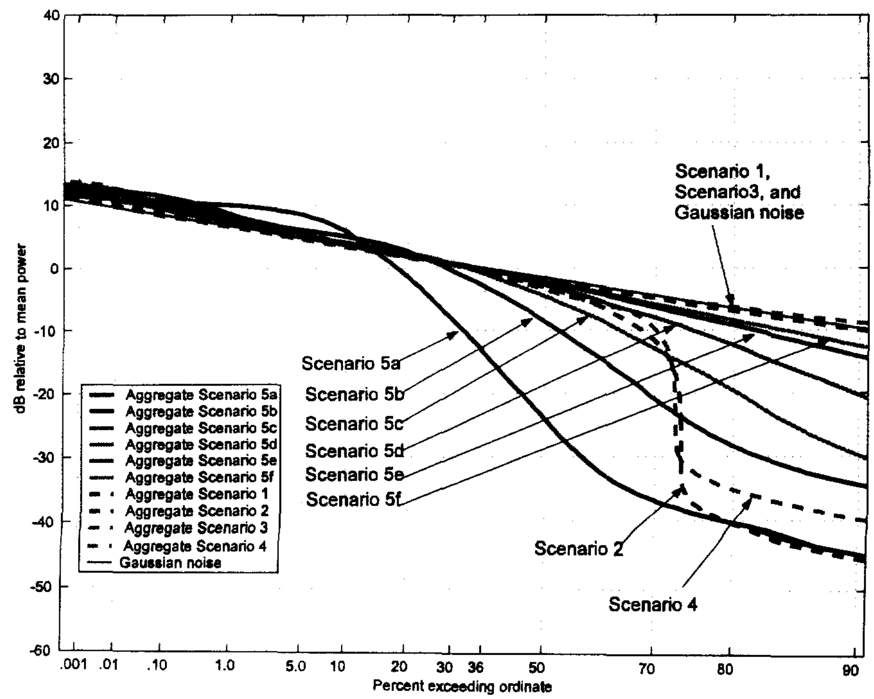

Figure C.3.30. APDs of aggregate UWB signals measured in a 3-MHz bandwidth.

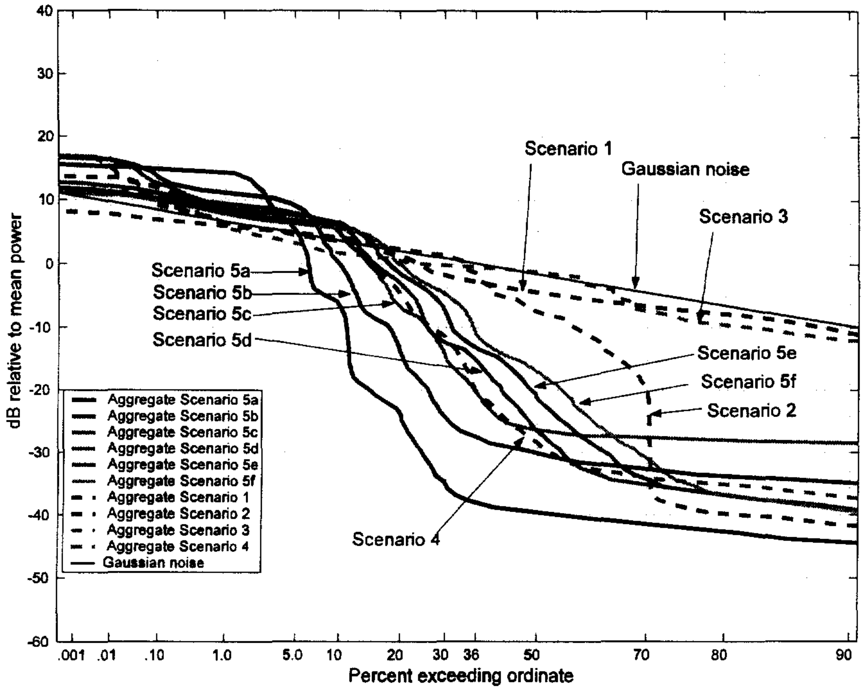

Figure C.3.31. APDs of aggregate UWB signals measured in a 20-MHz bandwidth.

APPENDIX D: THEORETICAL ANALYSIS OF UWB SIGNALS USING BINARY PULSE-MODULATION AND FIXED TIME-BASE DITHER

The theoretically derived spectra for fixed time-base dithered and binary pulse-modulated UWB signals were used to validate some of the test procedures described in this report. The calculated power spectral densities and examples showing how the analytical results compare to measurements are given below. The analytical results presented in this section are taken from [1].

Fixed time-base dithered UWB systems utilize short duration pulses transmitted at some nominal pulse period T. In this scheme, pulses are dithered about integer multiples of T. In the following discussion, it is assumed that the dither times are random variables θ_n that are independent and identically distributed over a fraction of the nominal pulse period with probability density $q(\theta_n)$. UWB signals may also include information bits by using binary pulse modulation in addition to the pulse dithering.

The power spectral density for a binary pulse-modulated fixed time-base dithered UWB signal is obtained by taking the Fourier transform of the autocorrelation function. Due to the periodic nature of the underlying pulsed signal, the process is *cyclostationary* with period T. Time averaging the autocorrelation function over a period yields the average power spectrum which depends only on the relative time delay. The time averaged power spectral density for a fixed time-base dithered UWB signal with binary pulse modulation is

$$\begin{aligned}
&\bar{R}_{xx}(f) = L + C \\
&L = \frac{1}{T^2}\left|\sum_{k=0}^{1} g_k \, P_k(f)\right|^2 \left|Q(f)\right|^2 \sum_n \delta(f - n/T) \\
&C = \frac{1}{T}\left[\sum_{k=0}^{1} g_k \left|P_k(f)\right|^2 - \left|\sum_{k=0}^{1} g_k \, P_k(f)\right|^2 \left|Q(f)\right|^2\right]
\end{aligned}, \qquad \text{D.1}$$

where P_k is the Fourier transform of the signal pulse for the information bit having the value k, Q is the Fourier transform of probability density function that describes the dithering, and g_k is the probability that an information bit has the value k (e.g., g_o is the probability that an information bit is "0" and g_1 is the probability that the bit is a "1"). Note that L is discrete (i.e., spectral lines) and C is continuous.

D.1 Fixed Time-base Dither and Pulse Position Modulation

If the bit values are equiprobable (i.e., $g_k = \frac{1}{2}$) and the pulse representing a 1 is a time delayed version of the pulse representing a 0 (i.e., $p_1(t+\xi) = p_0(t) = p(t)$), the power spectral density becomes

$$\begin{aligned}
\bar{R}_{xx}(f) &= L + C \\
L &= \frac{1}{2T^2}\left|P(f)\,Q(f)\right|^2\left[1+\cos\left(2\pi\xi f\right)\right]\sum_n \delta\left(f - n/T\right) \\
C &= \frac{1}{T}\left|P_k(f)\right|^2\left(1 - \frac{\left|Q(f)\right|^2\left[1+\cos\left(2\pi\xi f\right)\right]}{2}\right)
\end{aligned} \qquad \text{D.2}$$

Note that the discrete and continuous components depend on both the pulse spectrum and $Q(f)$. When $Q(f) \rightarrow 1$ (negligible dithering) and the information bits do not change, the continuous spectrum disappears leaving only a line spectrum as would be expected for a simple periodic pulsed signal.

The results of an example calculation using Equation D.2 when q is uniformly and continuously distributed between 0 and $T/2$ is given below. For this example, the signal consists of a short-duration pulse, shown in Figure D.1.1, transmitted at a 20 MHz rate. In this and following examples, it is assumed that ξ is small in comparison to the dithering, so that the effects of information bit modulation are negligible over the frequency range of interest.

The power spectral density over a frequency range of 1-5000 MHz is shown in Figure D.1.2. The magnitude of the spectrum is normalized to the peak of the continuous distribution (at about 250 MHz). The Fourier transform of the density function for this example is $Q(f) = \mathrm{sinc}(\pi f T/2)$. This function has nulls at frequencies equal to $2k/T$ ($k = \pm1, \pm2, \pm3, \ldots$); hence the interval between discrete spectral lines is 40 MHz, as shown in the figures. For frequencies above about 40 MHz, the continuous spectrum is approximately the same as the pulse spectrum (i.e., $P(f)$).

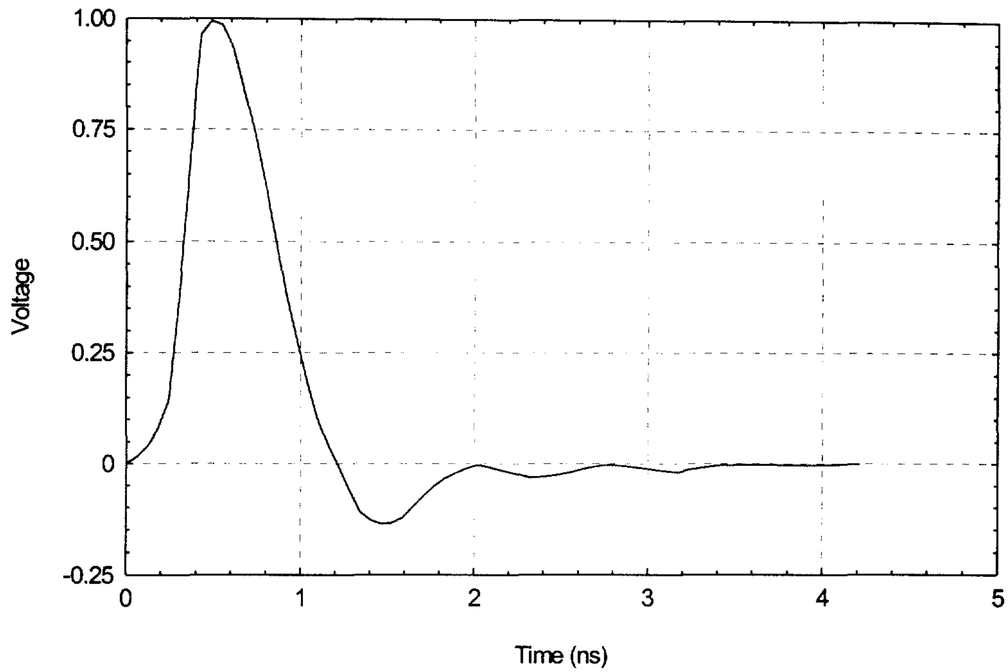

Figure D.1.1. Time-domain pulse shape.

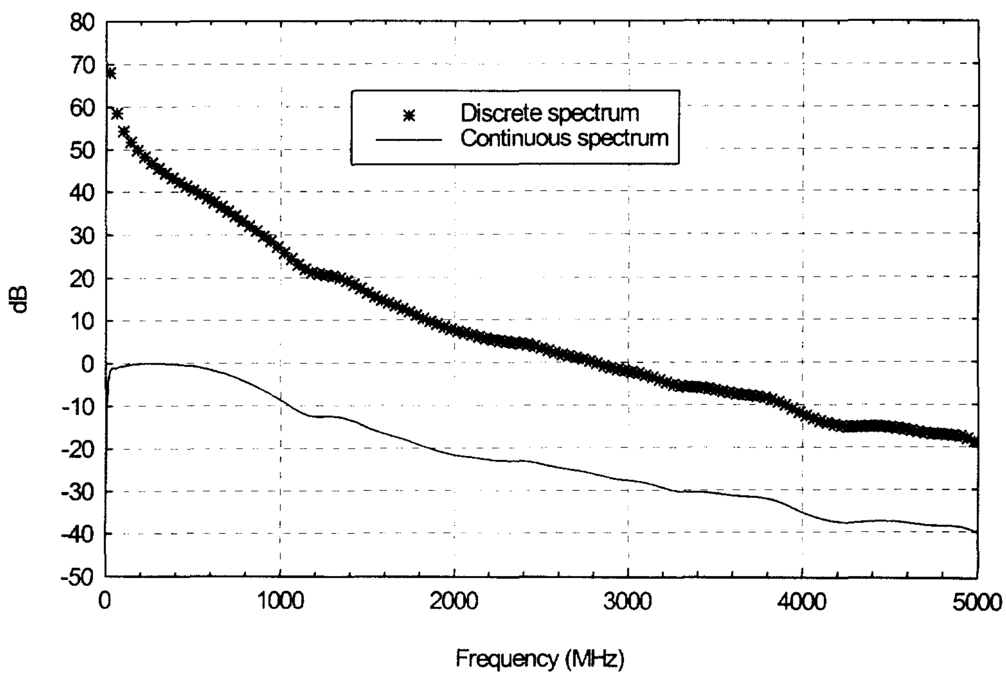

Figure D.1.2. Power spectral density for a fixed time-base dithered 10-MHz UWB signal. The pulse positions are continuously and uniformly distributed over 50% of the pulse repetition period.

The mean power in the bandwidth of a *narrowband* victim RF receiver as a function of frequency can readily be calculated from these results. For narrowband receivers where gains due to the UWB transmitter filters/antenna, propagation channel, and receiver are approximately constant over the receiver bandwidth, the received interference power can be calculated by applying the appropriate gain factors to the power in the receiver bandwidth at the center frequency of the receiver.

In the previous example, q is continuous and uniformly distributed over a fraction of the nominal period T. When the distribution is discrete so that the dithered pulse can only occur at particular times (e.g., $T - n\tau$, where $n = 0, 1, 2, 3, \cdots, N - 1$) with equal probability, the density function can be written as

$$q(t) = \frac{1}{N} \sum_{n=0}^{N-1} \delta(t - n\tau) \,, \qquad \text{D.3}$$

with spectrum

$$|Q(f)|^2 = \left(\sum_{n=-\infty}^{\infty} \operatorname{sinc}\left[\pi N \tau (f - n/\tau) \right] \right)^2 \,, \qquad \text{D.4}$$

which is a periodic function with period $1/\tau$. For example, when $1/T = 20$ MHz, and the pulse is discretely dithered over on half of the pulse-repetition interval with $\tau = 1$ ns, the spectrum is repeated at 1-GHz intervals as shown in Figure D.1.3. In this example, the receiver bandwidth is 1 MHz and the continuous spectrum is normalized to a maximum of 0 dB.

Note that for any integer m

$$|Q(m/\tau|^2 = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \,,$$

hence, the continuous spectrum decreases to a minimum at integer multiples of 1 GHz. For these frequencies, the discrete spectrum tends to a local maximum and spectral lines are significant. Contrast this with the case where q is continuous (i.e., $Q(f) = \operatorname{sinc}(\pi f T/2)$) described previously. With continuous dithering, spectral lines at multiples of 1 GHz are not present, since they occur at nulls of $\operatorname{sinc}(\pi f T/2)$.

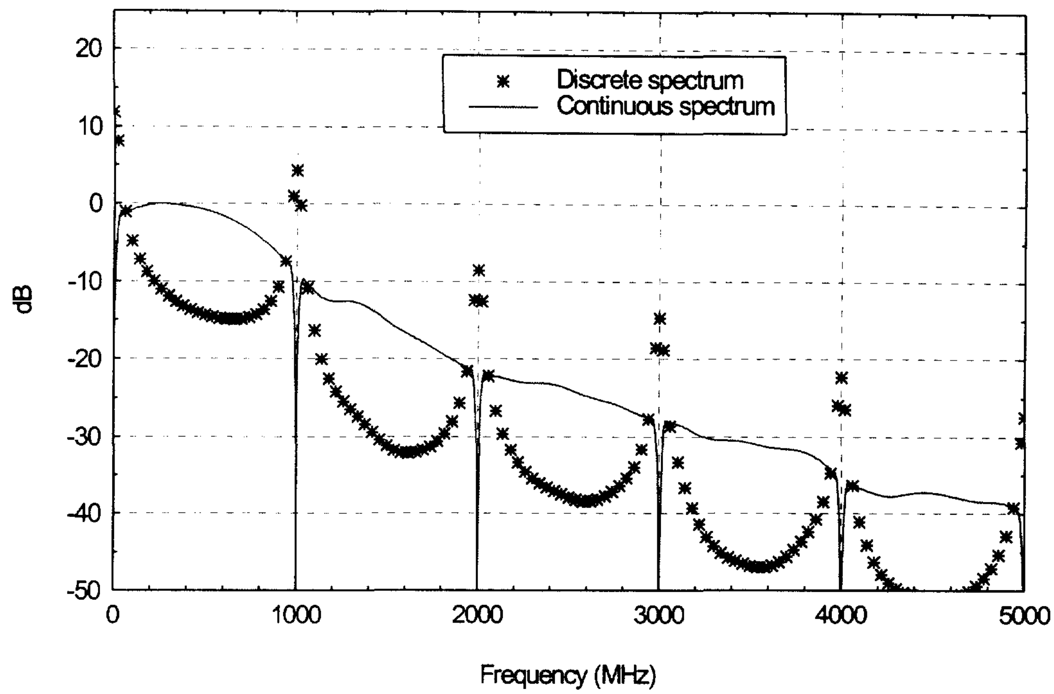

Figure D.1.3. Power spectral density for 20-MHz PRF 50% uniform discrete dithering with $\tau = 1$ ns.

A comparison of measured and predicted spectra for a discretely dithered UWB signal is shown in Figure D.1.4. The UWB signal is pulsed at a 20 MHz rate with uniform 50% discrete dithering with $\tau = 1$ ns. The measurement bandwidth is 1 MHz. As predicted, only three lines, the strongest at 1 GHz and two others at 1 GHz ± 20 MHz are visible in the measured signal. This figure shows good agreement between measurement and theory.

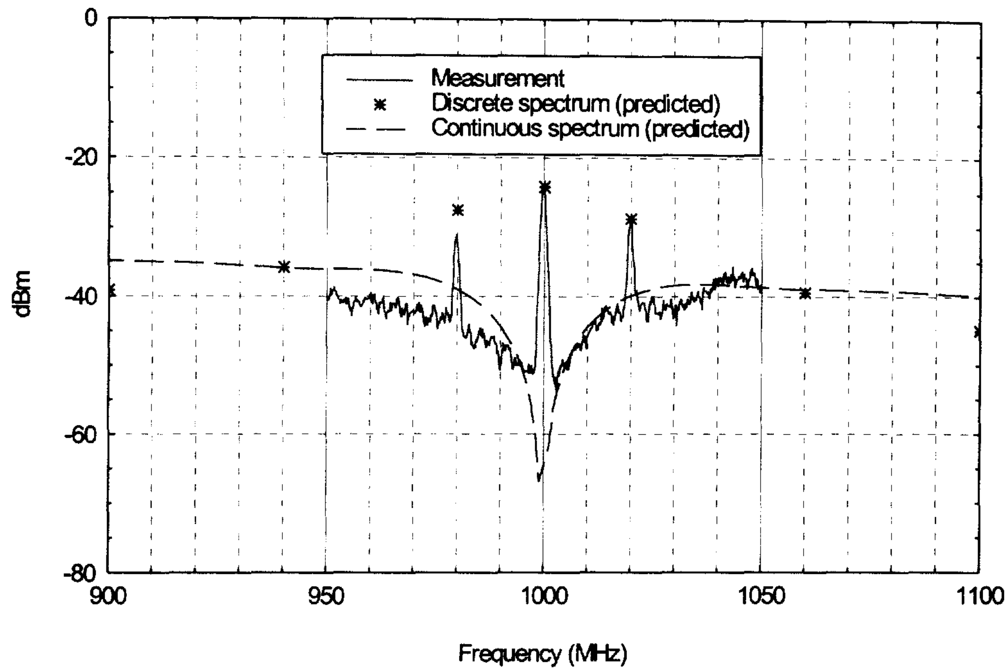

Figure D.1.4. Comparison of measured and predicted spectra for 20-MHz PRF 50% uniform discrete dithering with τ = 1 ns.

D.2 Power Spectrum for On-off Keying Without Dithering

For binary pulse modulation using on-off keying without dithering, we set $P_0 = P(f)$, $P_1 = 0$, $Q(f) = 1$ and $g_0 = g_1 = ½$ in Equation D.1 and obtain

$$\begin{aligned}\bar{R}_{xx}(f) &= L + C \\ L &= \frac{|P(f)|^2}{4T^2}\sum_n \delta(f - n/T) \\ C &= \frac{|P(f)|^2}{4T}\end{aligned} .$$

When the signal is passed through a narrowband receiver with center frequency f_c and the bandwidth B, the received power is

$$\int_{f_c - B/2}^{f_c + B/2} \bar{R}_{xx}(f)\, df = \frac{|P(f_c)|^2}{4T^2}[N + TB] ,$$

where N is the nominal number of lines in the filter passband. The ratio of the power in bandwidth B due to discrete and continuous components of the signal is simply $N(TB)^{-1}$.

Figure D.2.1 shows the spectrum of a signal generated by test equipment using on-off keying with equiprobable random bits and a pulse repetition frequency of 1 MHz. The signal was passed through a 20 MHz bandpass filter and a spectrum analyzer using a resolution bandwidth of B = 100 kHz. In this case N = 1, and hence, $(TB)^{-1}$ = 10 which is in agreement with Figure D.2.1 where the discrete spectrum is roughly 10 dB above the level of the continuous spectrum.

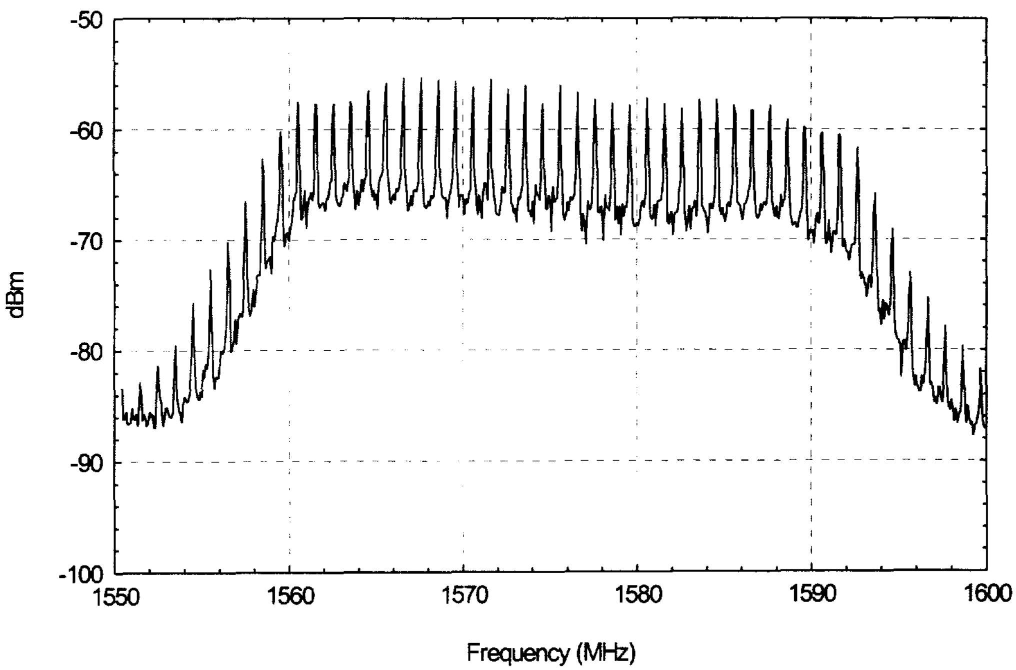

Figure D.2.1. Measured spectrum for on-off keying at 1-MHz PRF, B = 100 kHz.

References

[1] W.A. Kissick, Ed., "The temporal and spectral characteristics of ultrawideband signals," NTIA Report 01-383, Jan. 2001.

APPENDIX E. TUTORIAL ON USING AMPLITUDE PROBABILITY DISTRIBUTIONS TO CHARACTERIZE THE INTERFERENCE OF ULTRAWIDEBAND TRANSMITTERS TO NARROWBAND RECEIVERS

E.1 Introduction

The *amplitude probability distribution function* (APD) is used in radio engineering to describe signal amplitude *statistics*. The APD and its corresponding graph, shown in Figure E.1.1, succinctly express the probability that a signal amplitude exceeds a threshold. For example, the APD in Figure E.1.1 shows that the signal amplitude rarely exceeds high voltages. Statistics such as percentiles, deciles, and the median can be read directly from the APD. Other statistics such as average power can be computed with the APD.

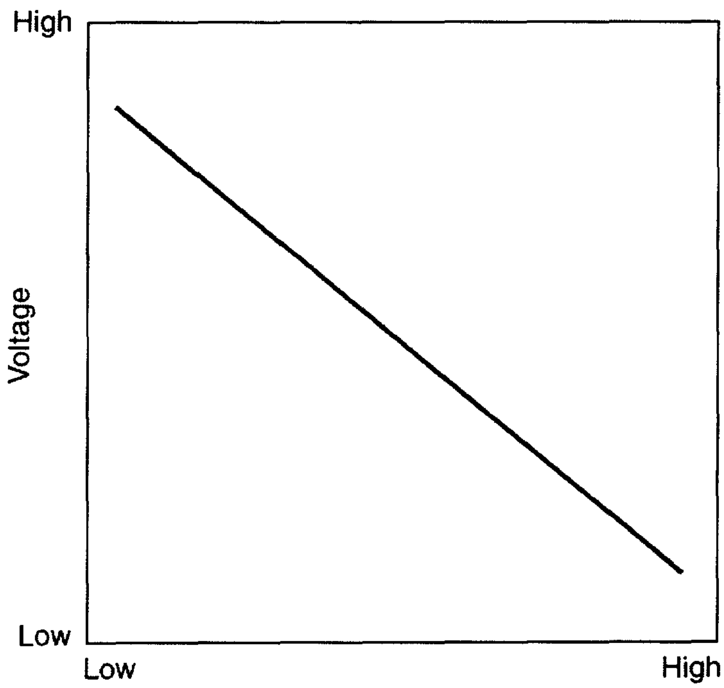

Figure E.1.1. Amplitude probability distribution.

The "signal" the APD characterizes is often noise or interference. For example, APDs are commonly used to characterize the amplitude statistics of *non-Gaussian* noise produced by lightning or unintentional emissions from man-made electrical or electronic devices. Numerous studies have shown that average noise power alone cannot predict the performance of receivers operating in non-Gaussian noise. APD statistics are needed for accurate predictions.

Today, many radio engineers are unfamiliar with the APD and its applications. This is because most modern receivers are designed to operate in bands with (zero-mean) *Gaussian* noise which is completely characterized by the average noise power statistic alone. Consequently, APD statistics are not needed for more accurate predictions.

Recently, Federal spectrum regulators have been asked to allow emissions from *ultrawideband* (UWB) transmitters to overlay bands licensed to services that use *narrowband* receivers. Critics have charged that UWB transmitters may cause interference to narrowband receivers. The amplitude statistics of this potential interference are dependent upon the specifications of the UWB signal and the band limiting filter in the narrowband receiver. The APD can be used to characterize this interference and correlate UWB signal and band limiting filter specifications to narrowband receiver performance.

The purpose of this tutorial is to introduce basic APD concepts to radio engineers and spectrum regulators who have not previously used the APD. It is hoped that these concepts will provide a firm basis for discussions on regulation of UWB transmitters. Emphasis is placed on understanding features likely to be found in band limited UWB signal APDs. These features are demonstrated with "tutorial" APDs of Gaussian noise, sinusoid (continuous wave) signals, and periodically pulsed sinusoid signals. Although the audience is intended to be broad, a limited number of mathematical expressions are used to avoid the ambiguity found in everyday language.

E.2 Signal Amplitude Characterization

E.2.1 APD Fundamentals

A *bandpass signal* is a signal whose bandwidth is much less than the *center frequency*. Bandpass signals are expressed mathematically as

$$s(t) = A(t)\cos(2\pi f_c + \theta(t)) \quad ,$$

where $A(t)$ is the baseband amplitude, $\theta(t)$ is the baseband phase, and f_c is the center frequency. The amplitude and phase define the *complex baseband signal*, $A(t)e^{j\theta(t)}$, whose spectrum is centered about 0 Hz.

The amplitude is always positive and is considered to be a *random variable*, A, when characterized by an APD. Formally, a new random variable, A_n, is present at each sampling instant. The set $\{A_1, A_2, \ldots A_N\}$ is called the *random sample* of the random variable A if each random variable is independent and identically distributed. Realizations or values of the random sample are denoted by the set $\{a_1, a_2, \ldots a_N\}$.

Associated with every random variable is a *probability density function* (PDF). The discrete PDF expresses the *probability* that a random variable "A" will have a realization equal to "a_i":

$$p(a_i) = P(A=a_i) \quad ,$$

where $P()$ is the probability of its argument. PDF values are positive and the area under a PDF is equal to 1.0.

The *cumulative distribution function* CDF expresses the probability that a random variable "A" will have a realization less than or equal to "a":

$$c(a) = P(A \le a) \quad .$$

The discrete CDF is obtained by integrating the discrete PDF

$$c(a) = \sum_i p(a_i) \quad ,$$

for all a_i less than or equal to a. CDF values range from 0.0 to 1.0 .

Radio engineers are generally more concerned about how often a noise or interference amplitude exceeds a threshold. Thus they often prefer to use the complement of the CDF (CCDF) or APD. The APD function expresses the probability that a random variable "A" will have a realization greater than "a":

$$cc(a) = P(A>a) \quad .$$

The discrete APD is obtained by subtracting the discrete CDF from 1.0:

$$cc(a) = 1.0 - c(a) \quad .$$

For clarification, Figure E.2.1.1 shows graphs of the discrete PDF, CDF, and APD for the random sample realizations:

$$\{a_1, a_2, \ldots a_{10}\} = \{1,2,3,3,1,4,4,3,4,3\} \text{ volts} \quad .$$

The discrete PDF is estimated from the histogram. By convention, the axes of the APD are oriented differently from the axes of the CDF and PDF.

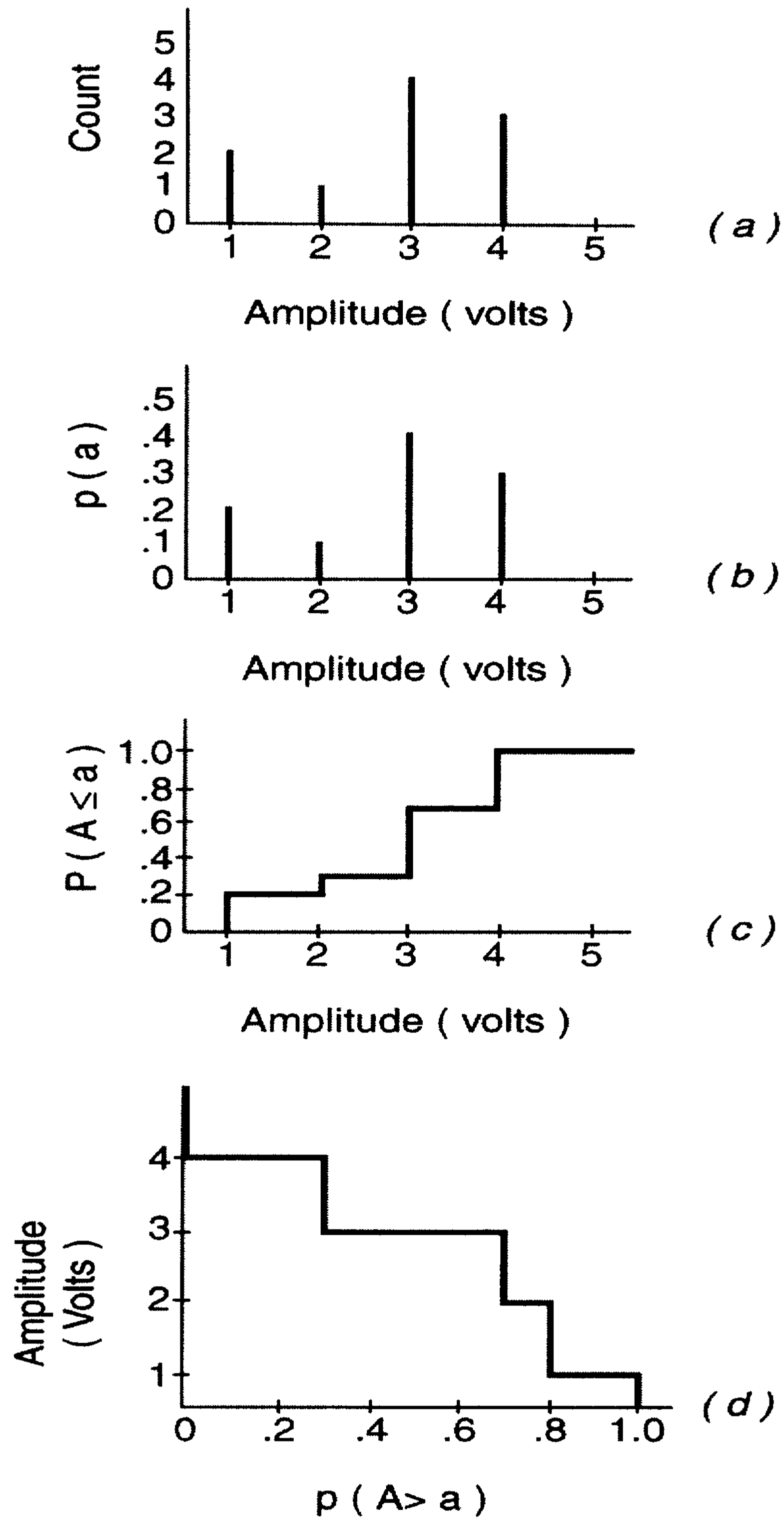

Figure E.2.1.1. Example histogram (a), probability density function (b), cumulative distribution function (c), and amplitude probability distribution function (d).

E.2.2 Statistical fundamentals

Statistics are functions that operate on the random sample. The *statistic value* is the result of a statistic operating on *random sample values*. Figure E.2.2.1 illustrates these relationships. Common statistical functions are percentile, mean or average, and root mean square (RMS). *First-order* statistics, addressed in this tutorial, assume the random variables are independent and identically distributed. *Second-order* statistics, not addressed in this tutorial, measure the correlation between these random variables. *Stationary* statistics are independent of time whereas *non-stationary* statistics are functions of time. Noise and interference amplitude statistics are non-stationary in many cases. Thus radio engineers sometimes measure the statistics of the amplitude statistics such as the median average noise power.

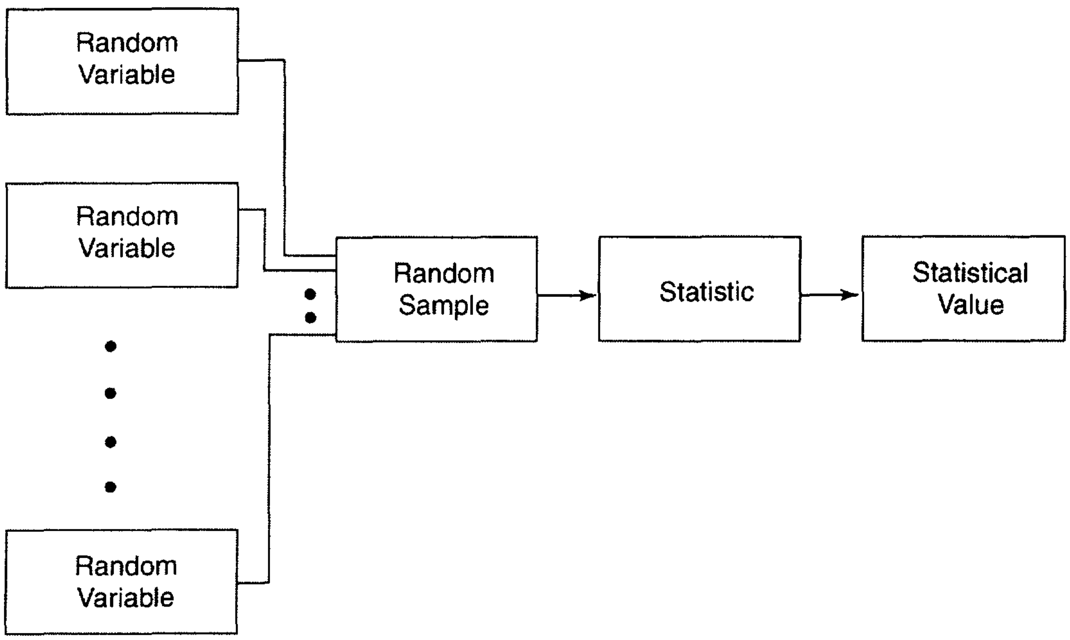

Figure E.2.2.1. Language of statistics.

Percentile amplitude statistics can be read directly from the APD. Peak and median amplitude statistics are the most widely used percentile statistics. The peak statistic is sometimes arbitrarily defined by the amplitude that is exceeded 0.0001% of the time:

$$V_p = cc^{-1}(0.000001) \quad ,$$

where $a = cc^{-1}(P(A>a))$. The median statistic is defined by the amplitude that is exceeded 50% of the time:

$$V_{median} = cc^{-1}(0.5) \quad .$$

The mean and RMS statistics are determined directly from the random sample values. The mean statistic is defined by:

$$V_{mean} = \frac{1}{N}\sum_{n} a_n \quad ,$$

where N is the number of samples. The mean-logarithm statistic is defined by:

$$V_{mean-\log} = \frac{1}{N}\sum_{n} \log_{10}(a_n) \quad ,$$

and the RMS statistic is defined by:

$$V_{RMS} = \sqrt{\frac{1}{N}\sum_{n} a_n^{\,2}} \quad .$$

The discrete APD and its corresponding discrete PDF can be used to calculate the mean and RMS statistics if the random sample values are no longer available. The choice of histogram bin size may affect the accuracy of these statistics. In this case the mean statistic is defined by:

$$V_{mean} = \sum_{i} a_i\, p(a_i) \quad ,$$

where a_i represents a discrete PDF value. The mean logarithm statistic is defined by:

$$V_{mean-\log} = \sum_{i} \log_{10}(a_i)\, p(a_i) \quad ,$$

and the RMS statistic is defined by:

$$V_{RMS} = \sqrt{\sum_{i} a_i^{\,2}\, p(a_i)} \quad .$$

As a reference, statistical values for the tutorial PDF presented in section E.2.1 are 4.0, 3.0, 2.8, 0.4, and 3.0 for the peak, median, mean, mean logarithm, and RMS statistics.

E.2.3 Graphing the APD

The APD of Gaussian noise is of particular interest to radio engineers because it is encountered in many practical applications. The amplitude of Gaussian noise is *Rayleigh distributed.* A Rayleigh distributed random variable is represented by a straight, negatively-sloped line on a *Rayleigh graph.* Figure E.2.3.1 shows the APDs of Gaussian and non-Gaussian noise on a Rayleigh graph.

The Rayleigh graph displays probability on the x-axis and amplitude on the y-axis. The probability is scaled by the function

$$x = 0.5\,\log_{10}(-\ln(P(A>a))) \quad ,$$

and converted to percent to represent the "*percent (of samples or time) exceeding ordinate.*" The amplitude in volts is converted to units of power such as dBW, i.e., scaled by the function

$$y = 20 \log_{10}(A) \quad ,$$

or alternatively it is displayed in dB relative to a standard noise power density or noise power.

Figure E.2.3.1 shows the statistics of Gaussian noise on a Rayleigh graph. Gaussian noise average power or RMS voltage corresponds to the power or voltage that is exceeded 37% of the time. Gaussian noise peak voltage is approximately 10 dB above RMS voltage. Gaussian noise average voltage, median voltage, and average-logarithm voltage are approximately 1dB, 2 dB, and 2.5 dB below RMS voltage respectively.

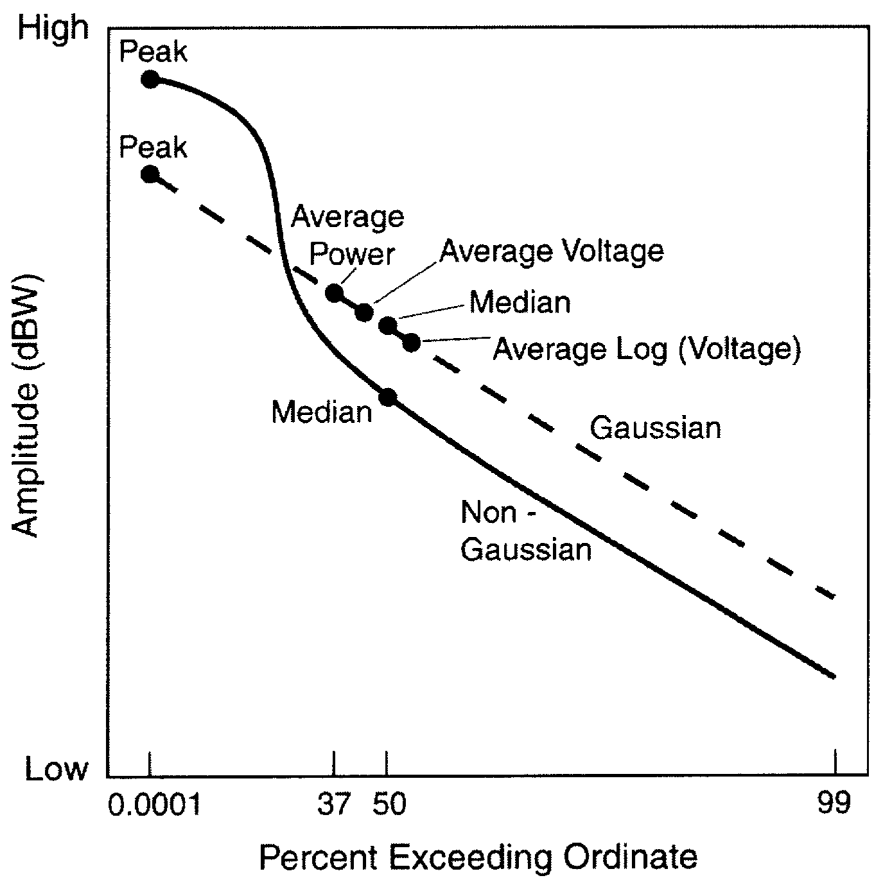

Figure E.2.3.1. Gaussian noise and non-Gaussian noise APDs plotted on a Rayleigh graph.

E.3 Tutorial APDs

E.3.1 Random Noise

Band limited random noise, i.e., the random noise present after a band limiting filter, is a random-amplitude and random-phase bandpass "signal" defined by

$$n(t) = A(t)\cos(2\pi f_c + \theta(t)) \ .$$

Band limited random noise is represented in the frequency domain by a *power spectral density* (PSD) in units of watts/Hz. The random noise "signal," amplitude, and PSD are shown in Figure E.3.1.1.

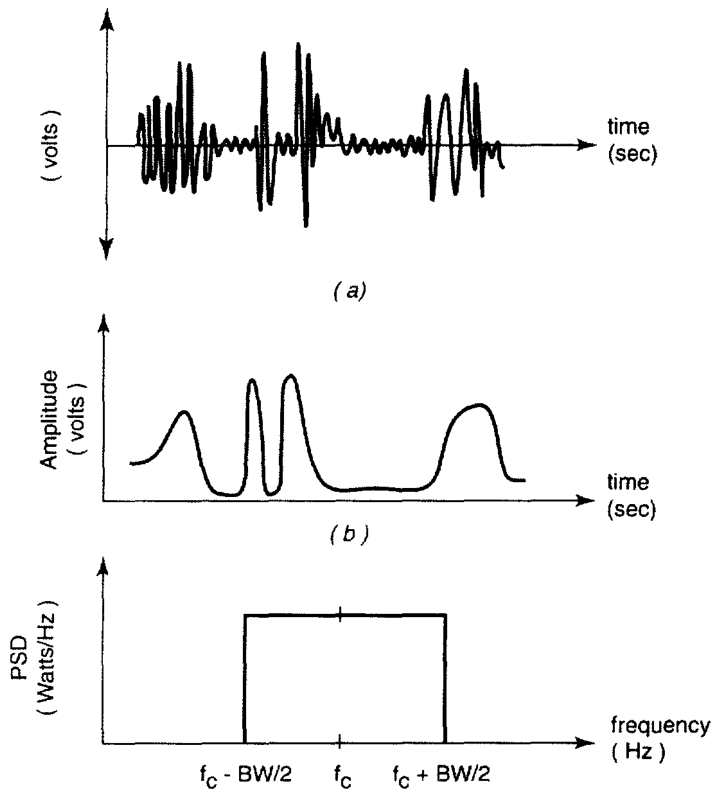

Figure E.3.1.1. Random noise (a), amplitude (b), and power spectral density (c).

Band limited noise average power is computed from the noise PSD:

$$P = \int_{f_c - BW/2}^{f_c + BW/2} N(f)df \quad ,$$

where $N(f)$ is the noise PSD in units of watts/Hz. Band limited *white* noise power density is constant over the band limiting filter bandwidth. As a result, the average noise power is directly proportional to the filter bandwidth and the RMS amplitude is directly proportional to the square root of bandwidth. This is sometimes referred to as the "10 $\log_{10}$ bandwidth" rule. The amplitude of band limited Gaussian noise is Rayleigh distributed. Figure E.3.1.2 shows the APD of band limited white-Gaussian noise (WGN) for two different bandwidths on a Rayleigh graph.

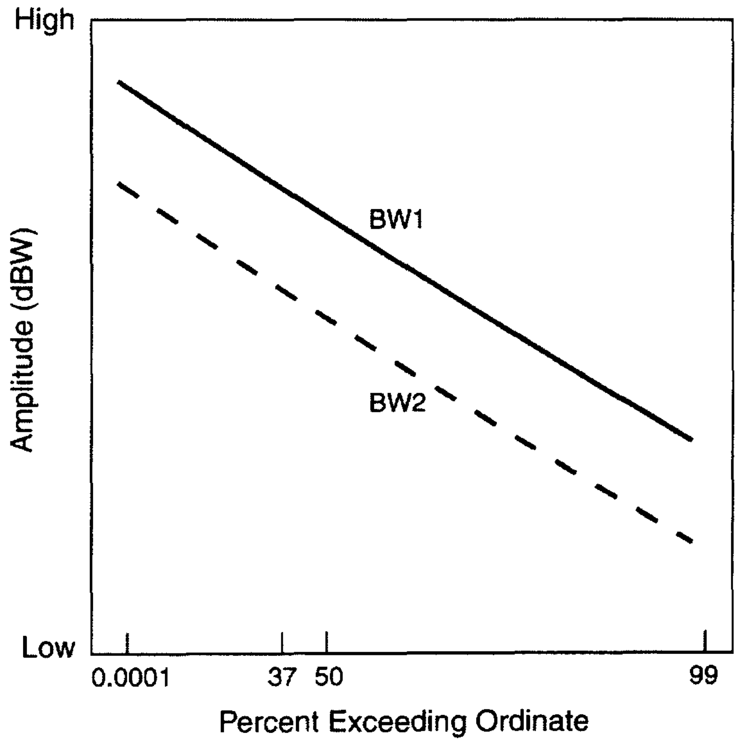

Figure E.3.1.2. Bandlimited Gaussian noise APDs with two different bandwidths. BW1 is greater than BW2.

E.3.2 Sinusoidal Signal

The sinusoid (continuous wave) signal is a narrowband, constant amplitude and constant phase signal. It is defined by

$$s(t) = A\cos(2\pi f_c + \theta) \quad .$$

The signal, signal amplitude, and amplitude spectrum are shown in Figure E.3.2.1. The APD of the sinusoid signal is a flat line from the lowest to the highest percentile on a Rayleigh graph. Changing the receiver center frequency can change the amplitude of the sinusoidal signal.

Widening the bandwidth of a receiver filter in the presence of noise causes the statistics to be *Rician*. Rician statistics are dependent on the ratio of the sinusoid power to the noise power. The Rician APD corresponds to the sinusoid signal APD when noise is absent and the Rayleigh APD when the signal is absent. Sinusoid, Rician, and Rayleigh APDs are depicted in Figure E.3.2.2.

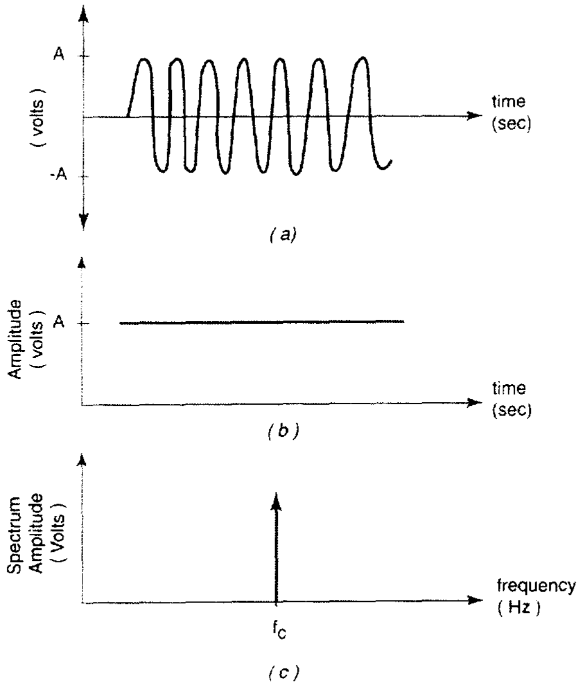

Figure E.3.2.1. Sinusoid signal (a), signal amplitude (b), and amplitude of the signal spectrum (c).

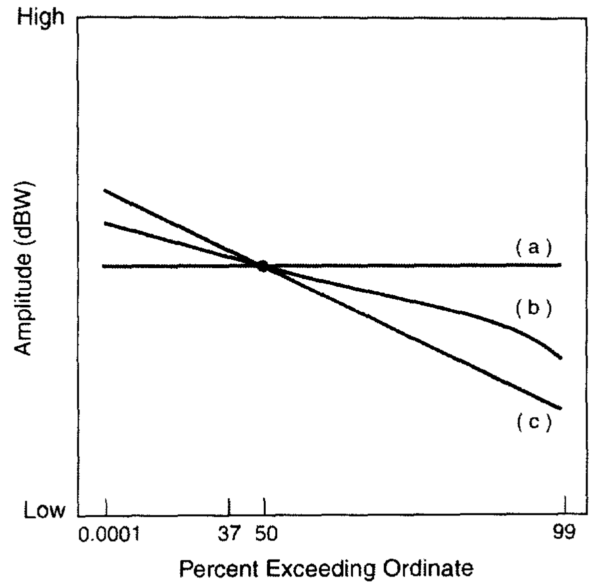

Figure E.3.2.2. Sinusoid signal APD without (a) and with (b) random noise. Sinusoid signal with random noise has Rician amplitude statistics. The Gaussian noise APD (c) is included for reference.

E.3.3 Periodically Pulsed Sinusoid

The periodically pulsed sinusoid is a deterministic, time-varying amplitude and constant phase signal defined by

$$s(t) = A(t)\cos(2\pi f_c + \theta) \quad .$$

The amplitude varies between 'on' and 'off' pulse states. The 'on' duration is the *pulse width* (PW) and the repetition rate of pulses is the *pulse repetition rate* (PRR) or the pulse repetition frequency (PRF). Amplitude spectrum *lines* are spaced at the PRF. Amplitude spectrum *nulls* are spaced by the reciprocal of the PW. The signal, signal amplitude, and amplitude of the signal spectrum are shown in Figure E.3.3.1.